Complex Numbers

The Basics

Definition: $i = \sqrt{-1}$

Powers of *i*

The pattern: $i^0 = 1$ $\quad i^1 = i$ $\quad i^2 = -1$ $\quad i^3 = -i$

$i^4 = 1$ $\quad i^5 = i$ $\quad i^6 = -1$ $\quad i^7 = -i$

To determine the value of an arbitrary power of i, such as i^{247}

1 Divide the exponent by 4 and examine the remainder

e.g., for i^{247}: $247 \div 4 = 61$, remainder 3

2 Apply the remainder in the above calculation as a power of *i* to get the final value

e.g., for i^{247}: 247 ÷ 4 = 61, remainder 3 $\rightarrow$ $i^3 = -i$ $\rightarrow$ $i^{247} = -i$

Complex Number Formats

Standard (Rectangular) $z = a + bi$

Polar $z = r(\cos\theta + i\sin\theta)$

$r = \sqrt{a^2 + b^2}$, $\tan\theta = \frac{b}{a}$

Exponential $z = e^{r\theta i}$

Arithmetic

Addition & Subtraction

Add the real and imaginary parts separately

e.g., $(6 + 2i) + (4 - 5i) = 10 - 3i$

$(6 + 2i) - (4 - 5i) = 2 + 7i$

Multiplication

FOIL the two complex values

e.g., $(6 + 2i)(4 - 5i) = 24 - 30i + 8i - 10i^2$

$= 24 - 30i + 8i - 10(-1)$

$= 34 - 22i$

Division

You want to rationalize the denominator; multiply the top and bottom of the fraction by the conjugate of the denominator:

e.g., $\frac{6 + 2i}{4 - 5i} = \frac{6 + 2i}{4 - 5i} \times \frac{4 + 5i}{4 + 5i} = \frac{14 + 38i}{16 - 25i^2} = \frac{14 + 38i}{41}$

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