Testing Hypotheses

Hypotheses

Null vs Alternative Hypotheses

- *Null Hypothesis*, H_0, is the claim regarding the population statistic that we are examining.
  - It is stated using the following notation
    - H_0: characteristic = value
- *Alternative Hypothesis*, H_a, is the competing claim; we are trying to find evidence for this.
  - It will have one of three forms:
    - H_a: characteristic < value
    - H_a: characteristic > value
    - H_a: characteristic ≠ value

P-values and Proving H_a

- The *P-value* of a test result is the probability of that result occurring if H_0 is true
  - We compare this to an agreed-upon threshold (the *significance level*, α).

    P-value < α – we have convincing evidence that H_a is true.

    P-value ≥ α – we have failed to disprove H_0 and we cannot take H_a as true.

    Note that we have *not* proven H_0; we have simply been unable to disprove it.

Type 1 & Type 2 Errors; Power

- *Type 1 Error* - Reject H_0 when H_0 is actually true
  - *i.e.,* falsely accept H_a

    The significance level, α, is the probability of getting a type 1 error.
- *Type 2 Error* - Not rejecting H_0 when H_0 is actually false
  - *i.e.,* falsely reject H_a

    β is the probability of getting a type 2 error
- *Power* - The probability that a test will correctly confirm H_a when H_a is, in fact, true.
  - Power $= 1 - \beta$

Testing a Population Proportion, π

Calculating a P-value

- We calculate the *P-value* of a test result by calculating the z-score of our result, compared to the hypothesized value, and then finding the probability associated with this z-score.
- The *test statistic* (z-value) of our test result is:

$$z = \frac{\text{statistic} - \text{parameter}}{\text{stddev of statistic}} = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Z - Z score/test statistic; p - proportion in sample
p_0 - hypothesized proportion; n - sample size

Validity requirements

This equation is valid if:

- Random data
- 10% rule
  $n \leq 0.1\,N$
- Large counts
  $n \cdot \hat{p} \geq 10$
  $n(1-\hat{p}) \geq 10$

- H_0 should be rejected if the P-value $\leq \alpha$
  - α is the significance level chosen for the test.

Obtaining P-value from z score

- *Upper-tailed test* — $H_\alpha: \pi > h$ — $P\text{-value} = 1 - P(z)$
- *Lower-tailed test* — $H_\alpha: \pi < h$ — $P\text{-value} = P(z)$
- *Two-tailed test* — $H_\alpha: \pi \neq h$ — if $z > 0$, $P\text{-value} = 2(1 - P(z))$
  if $z < 0$, $P\text{-value} = 2P(z)$

π - Test value from H_0; h - hypothesized value;
z - Z score/test statistic; P(z) - P-value from z table

Calculator Note

On your graphing calculator, three functions are associated with testing hypotheses:

- *1-PropZTest* — Proportion test
- *T-Test* — Mean (s_x)
- *Z-Test* — Mean (σ)

Testing a Population Mean, μ

Calculating the test statistic

The test statistic is a t score when testing a population mean.

$$t = \frac{\text{statistic} - \text{parameter}}{\text{stddev of statistic}} = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

t - t-score/test statistic; $\bar{x}$ - proportion in sample
s_x - sample standard deviation; n - sample size

Validity requirements

This equation is valid if:

- Random data
- 10% rule
  $n \leq 0.1\,N$
- Large counts
  $n \geq 30$