Vectors

Vector Notation

In printed material, vector names are indicated by heavy text: **A**.
In handwriting, vectors are usually indicated by an arrow above the name: $\vec{A}$

A = (r, θ)	Vector with specified r and θ values
B = $<x, y>$	Vector with specified x- and y-components
C = $x\mathbf{i} + y\mathbf{j}$	Vector with specified unit-vector components
$\lVert\mathbf{A}\rVert$ or $[\![\mathbf{A}]\!]$	Magnitude (*i.e.,* length) of vector **A**.

Basic Operations

In the following, $\mathbf{A} = <a_1, a_2>$, $\mathbf{B} = <b_1, b_2>$

Addition, subtraction, scalar multiplication

$\mathbf{A} + \mathbf{B} = <a_1 + b_1, a_2 + b_2>$

$\mathbf{A} - \mathbf{B} = <a_1 - b_1, a_2 - b_2>$

$c\mathbf{A} = <ca_1, ca_2>$ ← *c* is a scalar constant (a.k.a., a number).

Dot Product

The dot product of two vectors is a scalar value, *i.e.,* a number, not a vector.

- $\mathbf{A} \bullet \mathbf{B} = a_1 b_1 + a_2 b_2$
- $\mathbf{A} \bullet \mathbf{B} = \lVert\mathbf{A}\rVert\,\lVert\mathbf{B}\rVert\cos(\theta)$ ← θ is the angle between the two vectors

Other Vector Calculations

Vector between two points

Given points $A(x_1, y_1)$ and $B(x_2, y_2)$,

- Vector **AB** = $(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$

Unit Vector

$\hat{\mathbf{a}}$, the unit vector parallel to $\mathbf{A} = <a_1, a_2>$, is

- $\hat{\mathbf{a}} = \frac{a_1}{\lVert\mathbf{A}\rVert}\mathbf{i} + \frac{a_2}{\lVert\mathbf{A}\rVert}\mathbf{j}$

Orthogonal Vectors

If **A** and **B** are orthogonal (*i.e.* perpendicular to each other), then

$\mathbf{A} \bullet \mathbf{B} = 0$

Projection of a onto b

This is the component of **a** in the direction of **b**

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \times \mathbf{b}$$

Cross Product

The cross product results in a new vector that is perpendicular to the original vectors. Note that the cross product is applicable only to three-dimensional vectors.

- *Magnitude:* $A \times B = \|A\| \, \|B\| \sin(\theta)$
- *Direction:* perpendicular to the original vectors using the right-hand rule for $A \to B$

Formal Definition of cross product

Given vectors

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$
$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

Note this is the determinant of the matrix

- $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$