Conic Sections in Polar Coordinates

Conversions

Rectangular → Polar

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

Polar → Rectangular

$$r^2 = x^2 + y^2$$
$$\tan(\theta) = \frac{y}{x}$$

Conic Sections

Alternative Definition of a Conic Section

The set of points whose distance from a fixed point (the *focus*) and distance to a fixed line (the *directrix*) is a constant ratio.

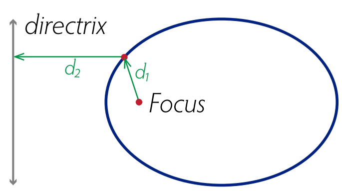
- The constant ratio is the *eccentricity* (e) of the curve; its value determines the type of conic.
  - $0 < e < 1$ Ellipse
  - $e = 1$ Parabola
  - $e > 1$ Hyperbola

Eccentricity and Conic Type

In the diagrams below, $e = \frac{d_1}{d_2}$

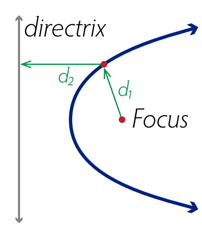
$0 < e < 1$

Ellipse



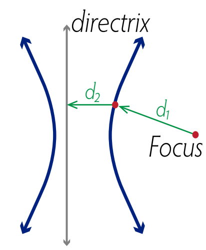
$e = 1$

Parabola



$e > 1$

Hyperbola



Polar Equations of Conic Sections

Vertical Directrix *(symmetric about polar axis)*

$$r = \frac{ep}{1 \pm e\cos\theta}$$

Horizontal Directrix *(symmetric about $\theta = \frac{\pi}{2}$)*

$$r = \frac{ep}{1 \pm e\sin\theta}$$

e = eccentricity; $|p|$ = distance between focus and directrix

General Equation of a Circle

The general polar equation for an arbitrary circle is:

$$r^2 - 2rr_0\cos(\theta - \phi) + (r_0)^2 = R^2$$

(r_0, ϕ) Center R Radius

Note that for a circle, $e = 0$