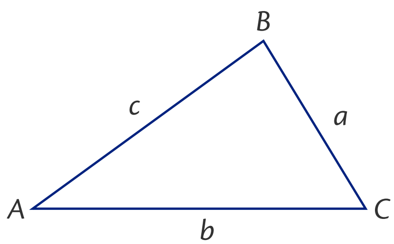
Precalculus Memorize Me™

Sines, Cosines, and Complex Numbers

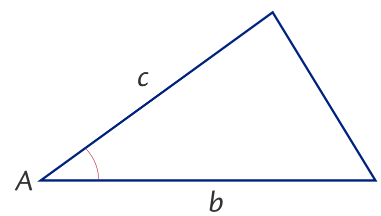
Sines

Law of Sines



$$\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$$

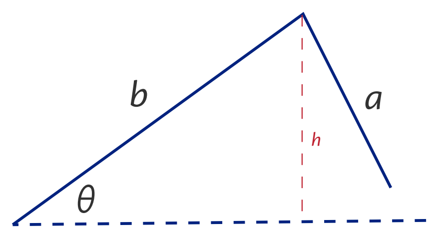
Area of an Arbitrary Triangle



$$\text{Area} = \tfrac{1}{2}\, bc \cdot \sin A$$

Law of sines, ambiguous case: Number of possible triangles

To determine the possible number of triangles:, given a triangle in which you know angle-side-side, as at right.



1 Calculate $h = b \sin \theta$

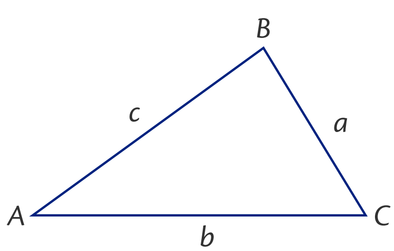
Note that b is the side adjacent to the angle

2 Compare h and a (the far side)

$h > a$	No triangle
$h = a$	One triangle (no ambiguity)
$h < a$	Two triangles

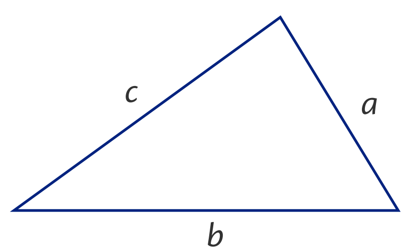
Cosines

Law of Cosines



$$a^2 = b^2 + c^2 - 2\,bc\cos A$$

Heron's Area Formula



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \tfrac{1}{2}(a + b + c)$

Trigonometric Form of Complex Numbers

Standard Form:

$z = a + bi$

Trigonometric form:

$z = r(\cos\theta + i\sin\theta)$

This is commonly abbreviated "$r\,\mathrm{cis}(\theta)$"

Also, by convention, should be on the interval $(0, \pi)$.

Converting from trigonometric to standard form:

$a = r\cos\theta$

$b = r\sin\theta$

Converting from standard to trigonometric form:

Draw a triangle for the complex number in the complex coordinate system.

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \frac{b}{a} \qquad \sin\theta = \frac{b}{r} \qquad \cos\theta = \frac{a}{r}$

Yes, this is the same as r, above

Absolute value, $|z| = \sqrt{a^2 + b^2}$

Multiplying and Dividing

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)] \qquad z_2 \neq 0$$

Powers and Roots *(De Moivre's Theorem)*

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right) \qquad k = 0, 1, 2, \ldots, n-1$$