Graphing Rational Expressions

What you need to do

- Completely factor the top and bottom of the rational expression.
- Collect the following information (preferably in this order):
  1. *Holes* – Look for same factor on top and bottom.
  2. *Vertical Asymptotes* - set the bottom equal to zero and solve for *x*.
  3. *Horizontal Asymptotes* - Look at the degree of the top and bottom (see below).
  4. *X-intercepts* - Set the top equal to zero and solve for x.
  5. *Y-intercepts* - Set *x* equal to zero and evaluate the function

Once you've cancelled out the common factor, you can do everything else using the reduced version of the expression.

Horizontal and Slant Asymptotes

The behavior of the graph as *x* goes to $\pm\infty$ (that is, the graph's horizontal or slant asymptotes) is indicated by the degrees (highest exponent) of the numerator ("top") and denominator ("bottom").

Horizontal Asymptotes

"Bottom heavy" *(degree bottom > top)*	Horizontal asymptote at y = 0.
"Balanced" *(degree bottom = top)*	Horizontal asymptote at y = ratio of the leading coefficients.
"top heavy" *(degree top > bottom)*	No horizontal asymptote (though there may be a slant (oblique) asymptote; see below.

Slant (Oblique) Asymptote

"top heavy by 1" *(degree top = bottom + 1)*

Do a long or synthetic division of the bottom into the top; the quotient (ignoring any remainder) is the equation of the slant asymptote.

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