Lines and Surfaces in Space

Lines and Planes

Line

For a line parallel to the vector $<a, b, c>$ and passing through point (x_0, y_0, z_0)

- *Parametric equations*

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + xt$$

- *Symmetric equations*

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Plane

$ax + by + cz + d = 0$

- $< a, b, c >$ is the plane's normal vector.

Basic Surfaces

The following assume the center of the surface is at the origin. If the center is at (x_0, y_0, z_0), replace x, y, and z, below, with $(x - x_0)$, $(y - y_0)$, and $(z - z_0)$.

Sphere

$x^2 + y^2 + z^2 = r^2$

Cylinder

Any equation of two quadratic variables. *e.g.*, $x^2 + y^2 = 4$

- The equation is the generating curve (directrix)
- The rulings are parallel to the missing axis.

Determining a Trace

To determine the nature of the trace of a surface on one of the coordinate planes, set the other variable to 0 and see what kind of conic remains.

Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- a, b, and c are the radii along the corresponding axis.

Hyperboloid, 1-sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

- The axis corresponds to the negative variable.

Hyperboloid, 2-sheet

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

- The axis corresponds to the positive variable.

Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

- The axis corresponds to the negative variable.

Elliptic Praboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- The axis corresponds to the linear variable.

Hyperbolic Paraboloid

$$z = \frac{y^2}{a^2} - \frac{x^2}{b^2}$$

- Saddle opens down along the linear variable.

Surface of Revolution

A surface of revolution is defined by two items:

- A two-dimensional *generating curve* (*e.g.*, $y = x^2$)
- An *axis of rotation* (*e.g.*, the x-axis)

The axis of rotation must be one of the axes taking part in the generating curve.

Equations

The equation of the surface is determined by the axis of rotation:

- x-axis $y^2 + z^2 = [r(x)]^2$
- y-axis $x^2 + z^2 = [r(y)]^2$
- z-axis $x^2 + y^2 = [r(z)]^2$

$r(x)$ is the generating curve.