Trigonometric Substitutions

Trig Substitution

The Substitutions

$\int \sqrt{a^2 - u^2}\, du$

Substitution: let $u = a \sin\theta$

Identity: $\cos^2\theta = 1 - \sin^2\theta$

Result: $\sqrt{a^2 - u^2}$ becomes $a\cos\theta$

$\int \sqrt{u^2 + a^2}\, du$

Substitution: let $u = a \tan\theta$

Identity: $\sec^2\theta = 1 + \tan^2\theta$

Result: $\sqrt{u^2 + a^2}$ becomes $a\sec\theta$

$\int \sqrt{u^2 - a^2}\, du$

Substitution: let $u = a \sec\theta$

Identity: $\tan^2\theta = \sec^2\theta - 1$

Result: $\sqrt{u^2 - a^2}$ becomes: $a\tan\theta$ for $u > a$

$-a\tan\theta$ for $u < -a$

Theorems ($a > 0$)

$\int \sqrt{a^2 - u^2}\, du$

$$\int \sqrt{a^2 - u^2}\, du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2}\right) + C \qquad u^2 > a^2$$

$\int \sqrt{u^2 - a^2}\, du$

$$\int \sqrt{u^2 - a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left|u + \sqrt{u^2 - a^2}\right|\right) + C \qquad u > a$$

$\int \sqrt{u^2 + a^2}\, du$

$$\int \sqrt{u^2 + a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} + a^2 \ln\left|u + \sqrt{u^2 + a^2}\right|\right) + C$$