

p- and Harmonic Series

- A **p-series** is a series of the form

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n^p} \quad \text{where } p > 0$$

For example,

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

- A **harmonic series** is a p-series with $p = 1$, i.e.,

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

- The **general form** for a harmonic series is

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{an + b} \quad (\text{note that } p \text{ is still equal to } 1)$$

p-Series Convergence

A p-series:

- **Converges** if $p > 1$
- **Diverges** if $0 < p \leq 1$.

Power Series

- A **power series** is a series of the form

$$F(x) = \sum_{n=0}^{\infty} a_n(x - c)^n$$

This is referred to as a "power series centered at c ."

where c is a constant and a_n is a coefficient derived from n

Note that if a_n is a constant and $c = 0$, a power function becomes a classic geometric series.

Interval of convergence

- The **interval of convergence** is the set of all x for which the power series converges.
- The **radius of convergence** is $\frac{1}{2}$ the size of the interval of convergence.
- Three possibilities:
 - Series converges only when $x = c$
 - Series converges for all values of x
 - Series converges absolutely when $|x - c| < R$, where R , the radius of convergence, is real and positive
- To find the interval of convergence, solve the ratio test for $L < 1$.
 - Must test the endpoints of the interval of convergence separately.
 - i.e., plug the endpoint x values into the original series and test the result for convergence.

Remember the Ratio Test?

Calculate

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- $L < 1$ Series converges
- $L > 1$ Series diverges
- $L = 1$ Inconclusive

Operations with Power Series

If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \sum_{n=0}^{\infty} b_n x^n$ then:

$$\blacktriangleright f(x^N) = \sum_{n=0}^{\infty} a_n x^{nN}$$

$$\blacktriangleright f(kx) = \sum_{n=0}^{\infty} a_n k^n x^n$$

$$\blacktriangleright f(x) \pm g(x) = \sum_{n=0}^{\infty} (a_n \pm b_n) x^n$$