# Definitions and Theorems

## Definitions

### Continuous

A function $f(x)$ is *continuous* at $x = c$ iff:

1 $\lim_{x \to c} f(x)$ exists

2 $f(c)$ exists

3 $\lim_{x \to c} f(x) = f(c)$

### *Differentiable*

A function is *differentiable* on open interval $(a,b)$ if $f'(x)$ exists for every value of $x$ on $(a,b)$.

- ▹ Alternative definition: $f(x)$ is differentiable at $x = c$ if $f'(x)$ is continuous at $x = c$.
- This means that $f(x)$ can't have a cusp or a vertical asymptote on $(a,b)$

### *Critical Numbers*

If $f(x)$ is defined at $x = c$, then $c$ is a *critical number* of $f$ if one of the following is true:

- $f'(c) = 0$
  - ▹ *i.e.,* $f(c)$ is a local maximum or minimum
- $f'(c)$ does not exist.
  - ▹ *i.e.,* $f(c)$ is a discontinuity or a cusp.

## Theorems

### Intermediate Value Theorem

If $f$ is continuous on $[a,b]$ and $k$ is between $f(a)$ and $f(b)$, then there exists a value $c$ on $[a,b]$ such that

$$f(c) = k$$

### Mean Value Theorem

If $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$, then there exists a value $c$ on $[a,b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- *In other words,* there will be some value of $x$ between $a$ and $b$ where the instantaneous slope of the function is equal to the average slope between $a$ and $b$.

### Rolle's Theorem

If $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$, and $f(a) = f(b)$, then there exists a value $c$ on $[a,b]$ such that

$$f'(c) = 0$$

- *In other words,* there will be some value of $x$ between $a$ and $b$ where the function has a local max or local min.

### Extreme Values Theorem

If $f$ is continuous on $[a,b]$, then $f$ has both an absolute maximum and absolute minimum on $[a,b]$.