Complex Numbers Summary

The Basics

Definition: $i = \sqrt{-1}$

Powers of *i*

The pattern: $i^0 = 1$ $i^1 = i$ $i^2 = -1$ $i^3 = -i$

$i^4 = 1$ $i^5 = i$ $i^6 = -1$ $i^7 = -i$

To determine the value of an arbitrary power of i, such as i^{247}:

1 Divide the exponent by 4 and examine the remainder

e.g., for i^{247}: $247 \div 4 = 61$, remainder 3

2 Use the remainder in the above pattern to get the final value

e.g., for i^{247}: our remainder was 3; $i^3 = -i$, so $i^{247} = -i$

Arithmetic

Addition & Subtraction

Add the real and imaginary parts separately

e.g., $(6 + 2i) + (4 - 5i) = 10 - 3i$

$(6 + 2i) - (4 - 5i) = 2 + 7i$

Multiplication

FOIL the two complex values

e.g.,
$$\begin{aligned}(6 + 2i)(4 - 5i) &= 24 - 30i + 8i - 10i^2 \\ &= 24 - 30i + 8i - 10(-1) \\ &= 34 - 22i\end{aligned}$$

Division

Multiply the top and bottom of the fraction by the conjugate of the denominator:

e.g.,
$$\frac{6 + 2i}{4 - 5i} = \frac{6 + 2i}{4 - 5i} \times \frac{4 + 5i}{4 + 5i} = \frac{14 + 38i}{16 - 25i^2} = \frac{14 + 38i}{41}$$